

# ***SPECTRA RESEARCH Institute***

Final Report

## Neuroelectric Activity and Analysis in Support of Direct Brainwave to Computer Interface Development

Richard H. Dickhaut

prepared for the Office of Naval Research under  
Grant # N00014-97-1-0926

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13. ABSTRACT (Maximum 200 words)

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**Neuroelectric Activity and Analysis in Support of Direct  
Brainwave to Computer Interface Development**

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**Abstract**

Electroencephalographic (EEG) scalp sensors were used as probes to acquire neuroelectric data correlated in real time with cognitive processing of linguistic thought stimuli. The adoption of a suitable data analysis methodology provides results that show a direct relationship with a characteristic, quantifiable set of neural activity for digits from zero to nine, or the words 'yes' or 'no'.

An experimental design was centered on timed capture of single epoch neuroelectric data with EEG sensors upon the initiation of thought stimuli. Experimental results were analyzed for three test subjects and twelve linguistic stimuli at two electrode locations. The waveform from each location and each single stimulus event was subjected to a decomposition procedure based on the premise that macrocolumns, acting as modular processing units, contribute their own simultaneous subwaveforms to a resultant composite waveform detected by the EEG sensor. The decomposition subwaveforms analytically obtained are the result of an exact solution of a set of equations applied to the experimental data for each individual epoch. Each decomposition subwaveform has its own distinct frequency, amplitude, phase, and decay characteristic. The analytical outcome yields parametric coefficients that determine the characteristics of each subwaveform.

The resultant set of parametric coefficients from a group of thirty single data epoch waveforms were used to generate a three dimensional parametric surface that is characteristic and specific for an individual digit or word at a given electrode location for each test subject. Two electrode locations were chosen that are associated with cognitive, linguistic processing. Each digit or word, then, has a set of 3-D neural activity maps corresponding to the two electrode locations.

## **Background**

How information is processed, represented, and actively used in the brain is a major question. When the eyes observe a written word, neural spike train sequences are set into play in nerves everywhere along the pathway from the eyes to layers of the neocortex. From there on, immense complexities challenge our capability to model the computational behavior of large groups of neurons densely interconnected with many thousands of synapses per neuron, and any emergence of large scale, complex system behavior. The notion that computation is involved is attractive because it is a reasonable hypothesis to assume the brain must somehow represent information for short time purposes as well as store it for future use, and electrical representation of information in a compact form achieved by computation is a familiar concept.

The general structural characteristics of the neocortex can supply the inspiration and a good basis for model construction. Neocortical neurons reside in vertical columns perpendicular to the cortical surface (Mountcastle, 1957,1978; Hubel and Wiesel, 1977; Nauta, 1979; Kaas et al., 1981; Gazzaniga, 1989), as well as in six horizontal layers. These columns have been referred to as macrocolumns, and two decades ago the suggestion was made that they might be modular processing units (Rakic, 1976).

Two 1996 papers supply useful insights that could support the idea of macrocolumns as modular processing units. Traub et al. (1996) describe interneuron networks that generate a distribution of 20 - 70 Hz oscillations arising from spike doublets dependent on intracellular current injection. Intragroup connections between networks result in coherent oscillations. The paper by Gray and McCormack (1996) introduces a subset of pyramidal cells in layers 2 and 3 of the visual cortex, called chattering cells (CH), whose stimulation from driving current injection results in doublet bursts. These experimentally observed 20 - 70 Hz oscillations, again in a distribution, result in synchronous firing between cells in different macrocolumns. The CH cells also receive oscillatory bursts in the same range from interneuron sources. The doublet bursts are an effective form of presynaptic input -- rapid firing can lead to postsynaptic temporal summation and increase dramatically the probability of presynaptic neuron transmitter release. CH cell projections provide a basis for generation of long range synchronous cortical activity.

The end surface area of the columns is small compared to the surface area of a probe of neuroelectric brain activity such as an EEG electrode. Suppose that the information output of each of these columns is more or less independent, but some redundancy is allowed among groups of closely associated columns that may be computing similar information content. Let the dynamic change in electrical charge per unit time in these groups of columns be sensed by the EEG probe over a cortical gyrus, and let there be minimal influence from weaker sources farther away. The result should be an EEG probe signal representing a composite electrical signature that reflects the summation of the signatures from each of the column groups more or less directly underneath the probe. In the case of the cognitive stimuli experiments being considered here, the signal response will be referred to as an event-related-potential (ERP).

For modeling purposes, a key ingredient is the selection of an appropriate analytical technique that can operate on the composite ERP signal and decompose the signature into its original set of separate waveforms. The technique that has been chosen and found sound in previously published research (Dickhaut, 1988) also had two other attributes that were added to the cognitive model considerations noted above, namely, that the experimental system output was produced by an impulse function response and that only pulse waveform phenomena would be extracted in the analysis. The first of these two attributes means that the brain, as the system in question, would be considered to be in an ambient ready state prior to the sudden appearance of a stimulus or the production of a thought, and would then have the capability to respond, as many physical systems would, to a single exciting stimulus (e.g., with peak amplitude and exponential decay phenomena, etc.). In the application of this idea to the brain, it is assumed likely that the stimulus response is subserved by real time dynamics of a mechanism akin to reentrant signaling and information processing as proposed by Edelman (1979). The second attribute means that all continuously valued wave phenomena with the same frequency and amplitude throughout the duration of the electrical signal would be ignored.

The particular waveform decomposition technique chosen presumes that the target ERP signature is the result of an excited structure responding to impulse phenomena, and is composed of multiple simultaneous waveforms each having its own structural frequency characteristic.

## **Results**

Experimental results were analyzed for three test subjects and twelve linguistic stimuli at two electrode locations that lie directly over cortical areas associated with cognitive processing. The two areas were Brodmann's 39/40 over the left hemisphere; and O<sub>2</sub> for the right hemisphere. Waveforms were recorded for the resultant neural activity associated with the thought of a digit ranging from one to nine or the thought of the words 'yes' or 'no', and subjected to a special decomposition analysis. For secondary comparison purposes, waveforms were also recorded and analyzed for the experimental condition in which the brain responded to the presentation on a computer screen of the same digits and words. For both experimental conditions, the waveform from each location and each single stimulus event was subjected to a decomposition procedure based on a model of an excited structure responding to impulse function neural driving currents that, in turn, were initiated by the sudden onset of the cognitively meaningful stimulus. A set of subwaveforms was produced by the decomposition procedure. Each subwaveform in the decomposition set has its own distinct frequency, amplitude, phase, and decay characteristic.

The next step in the process was to use the analytically-derived parameter coefficients from the subwaveforms to produce a 3-D topographical map from the data for each individual event (i.e., physiological voltage activity in the defined time epoch) and stimulus type, as well as electrode location, and human subject. The 3-D map comprises

Figure 1a. Subject 1, parietal, thought stimulus = 2

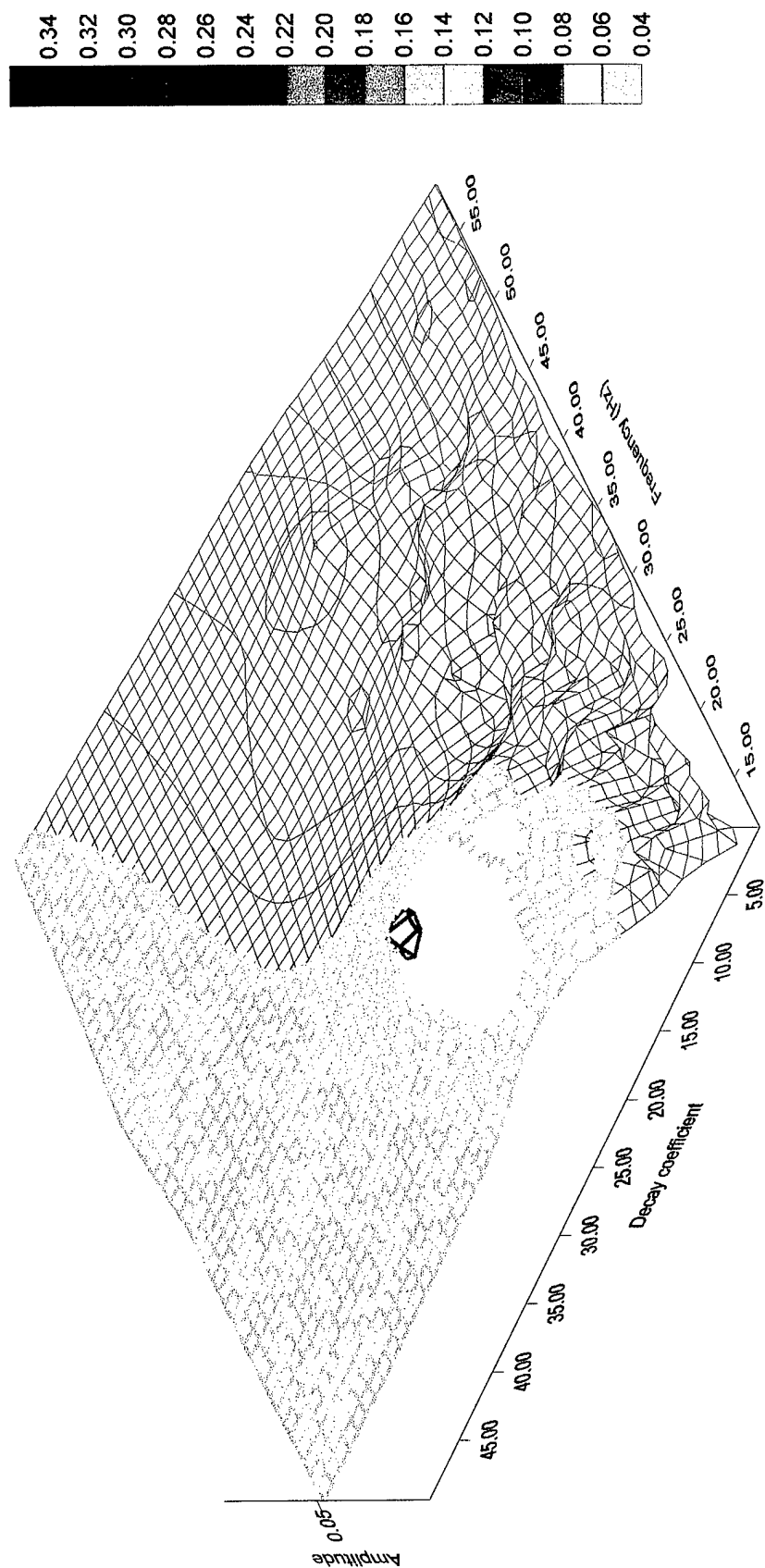


Figure 1b. Subject 2, parietal, thought stimulus = 2

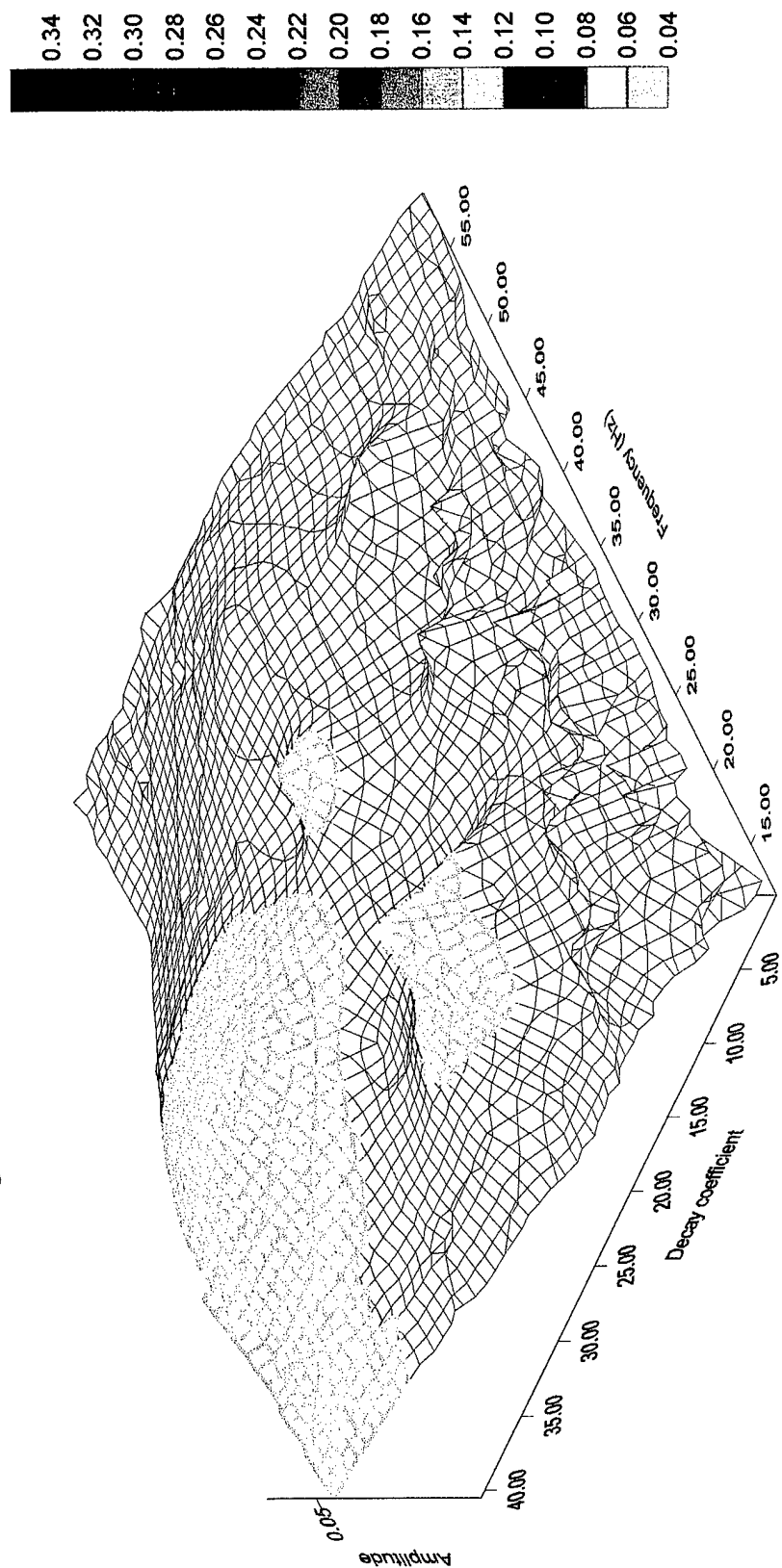




Figure 1c. Subject 3, parietal, thought stimulus = 2

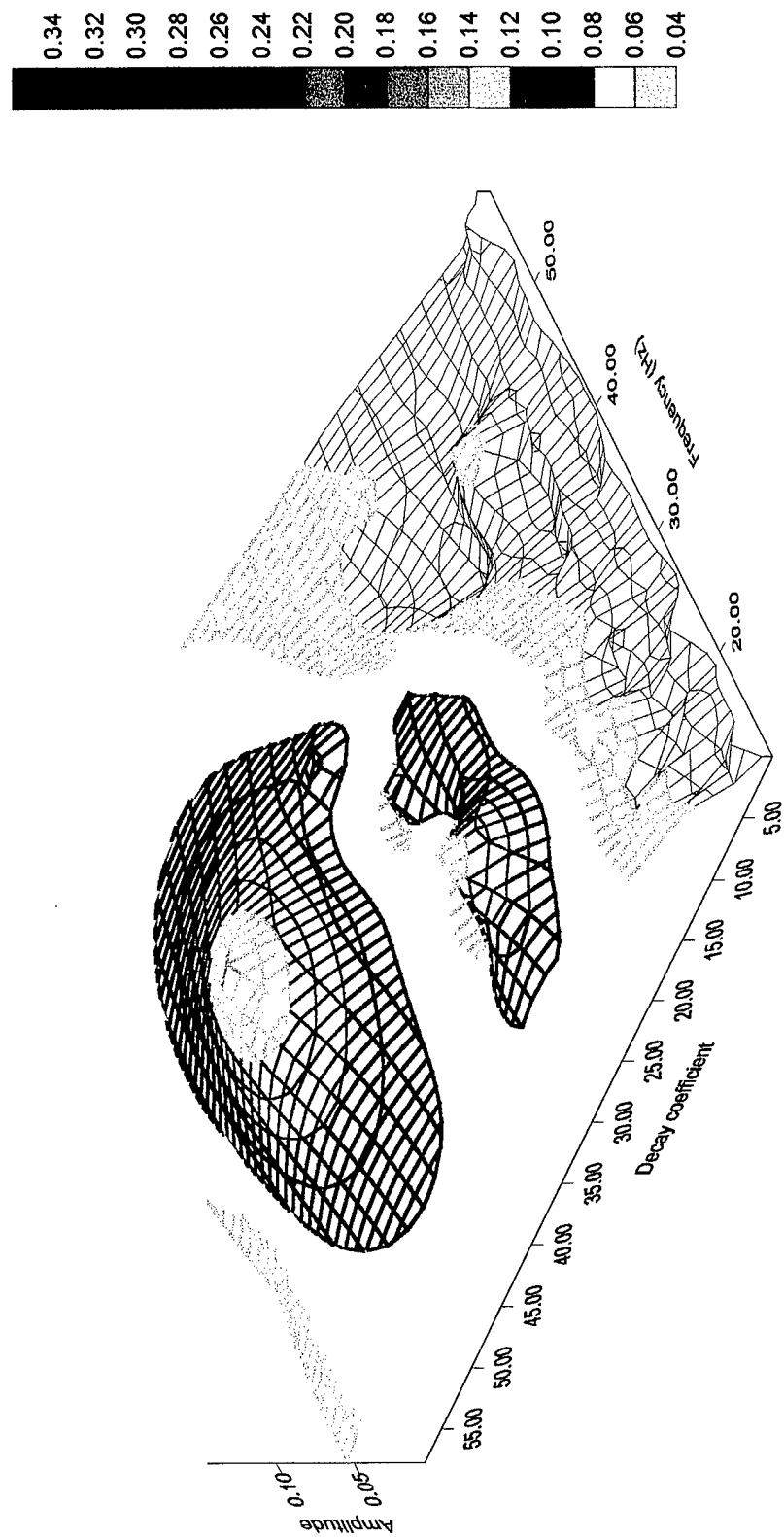


Figure 2a. Subject 1, parietal, thought stimulus = 9

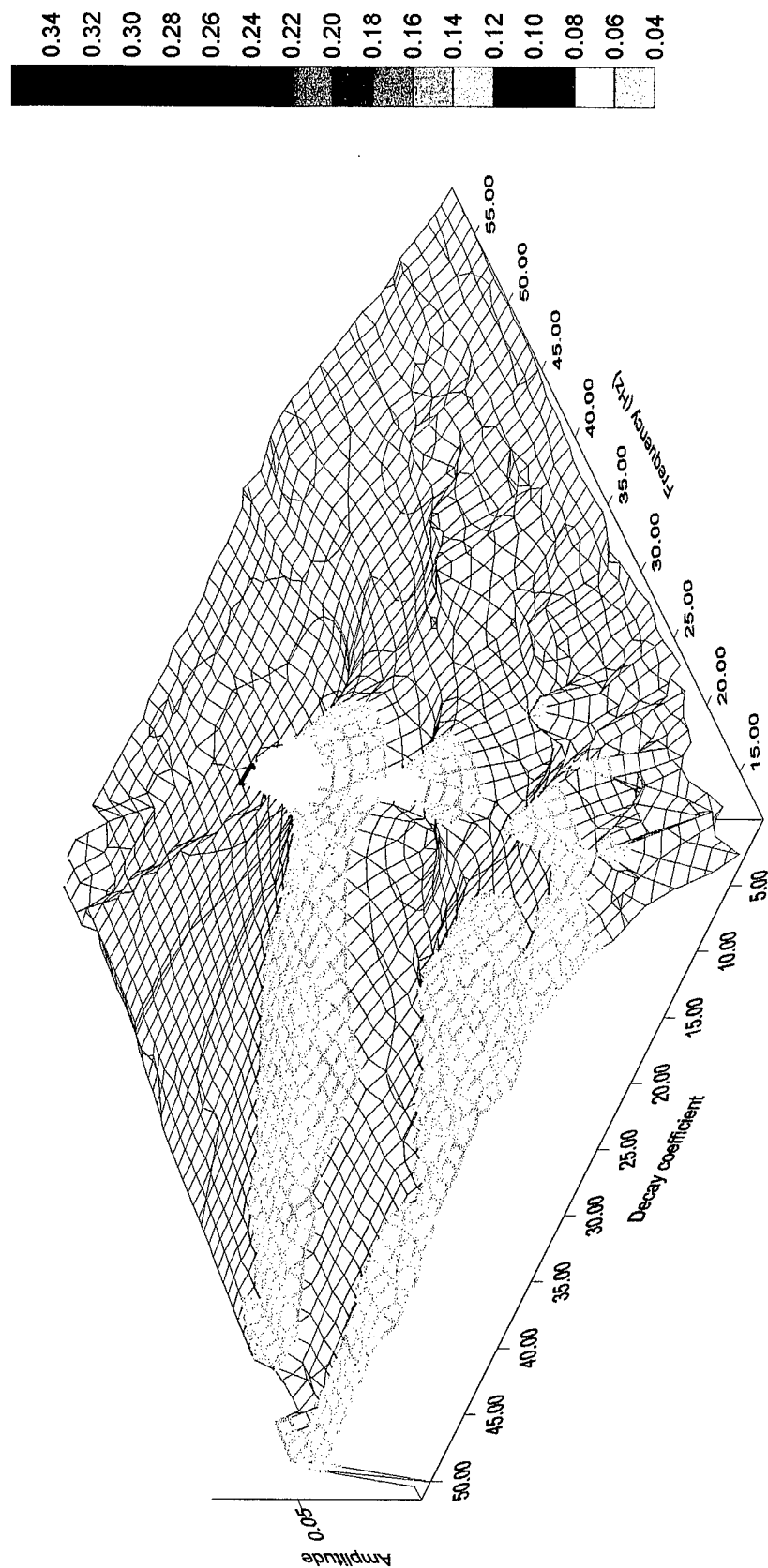


Figure 2b. Subject 2, parietal, thought stimulus = 9

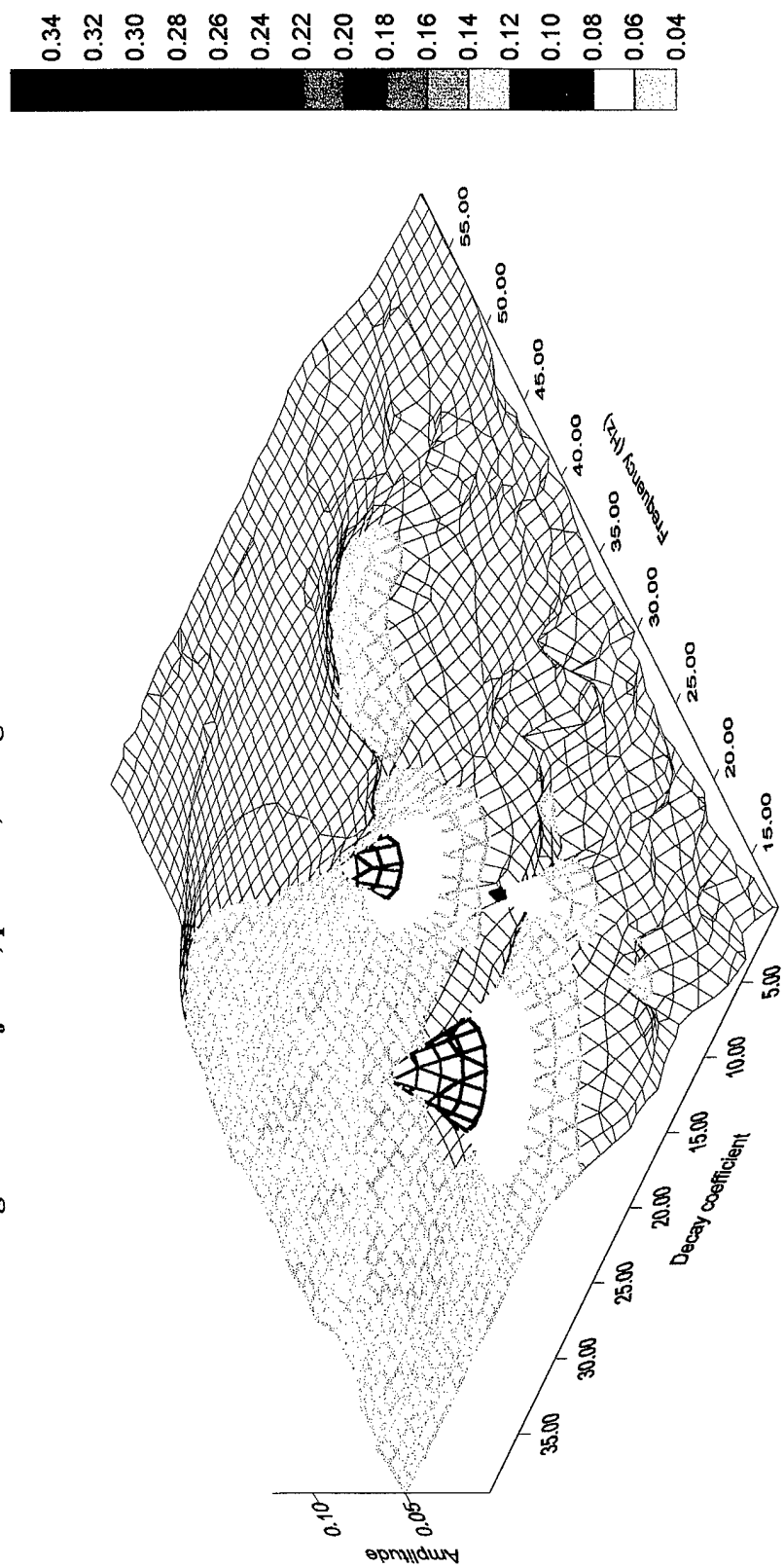


Figure 3a. Subject 1, occipital, thought stimulus = 9

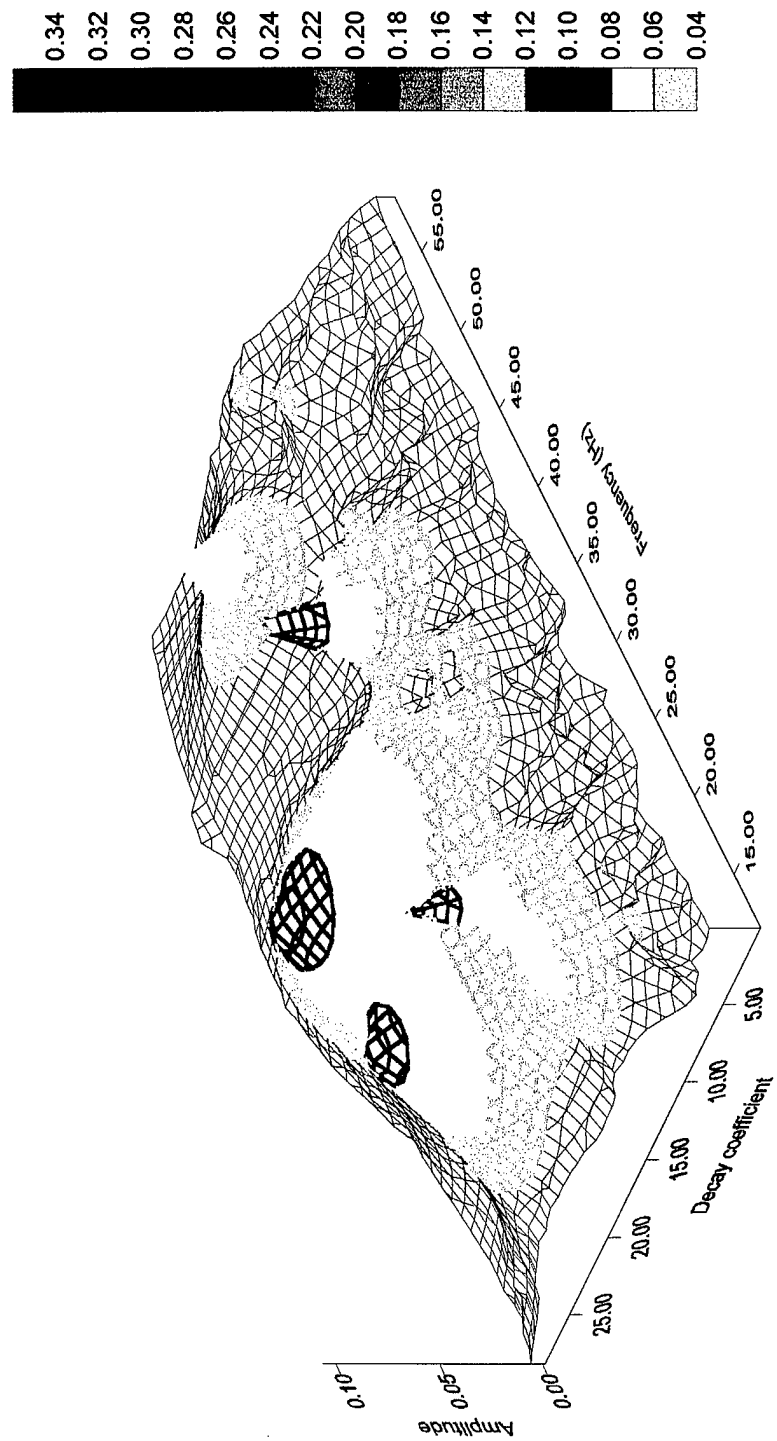


Figure 3b. Subject 2, occipital, thought stimulus = 9

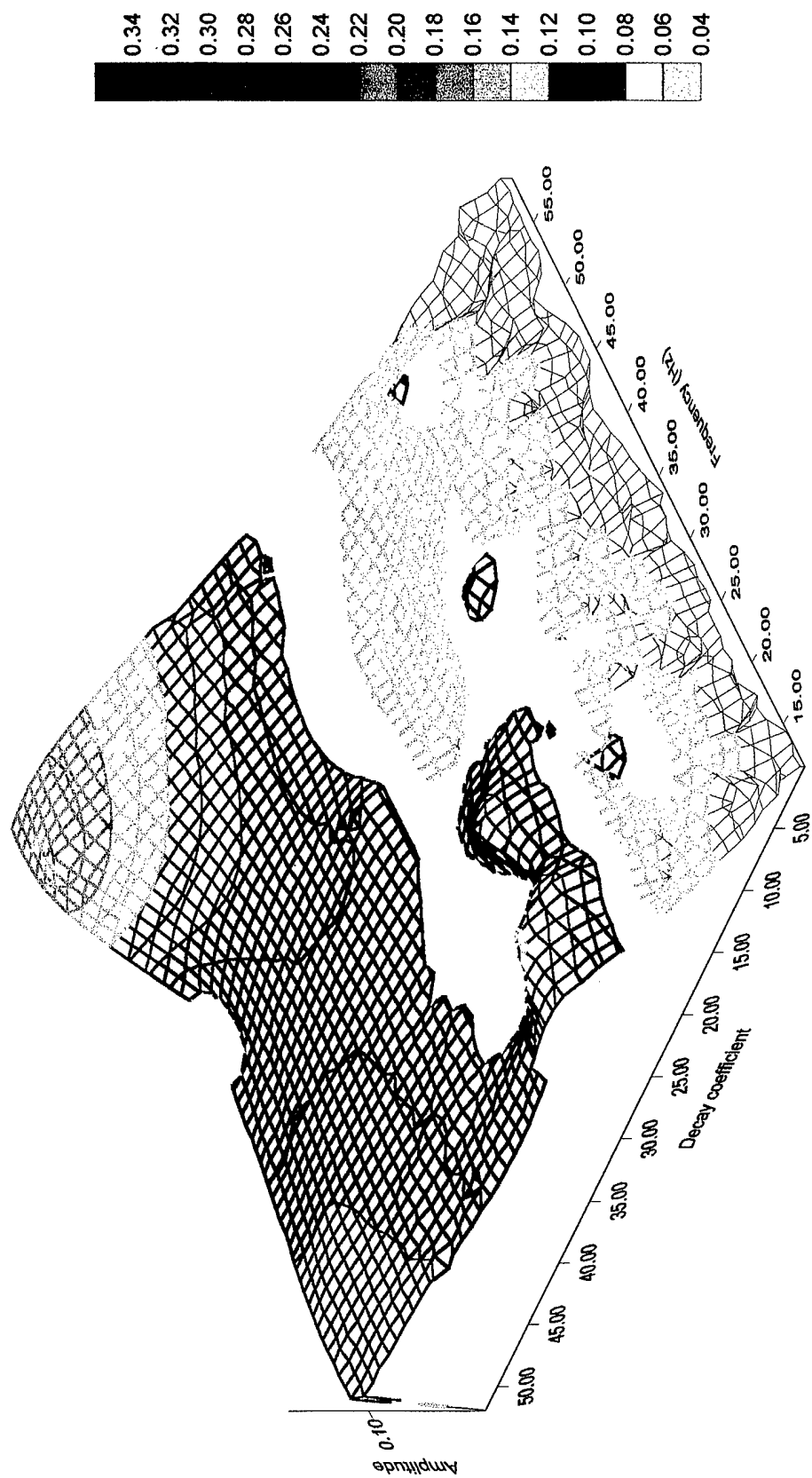


Figure 3c. Subject 3, occipital, thought stimulus = 9

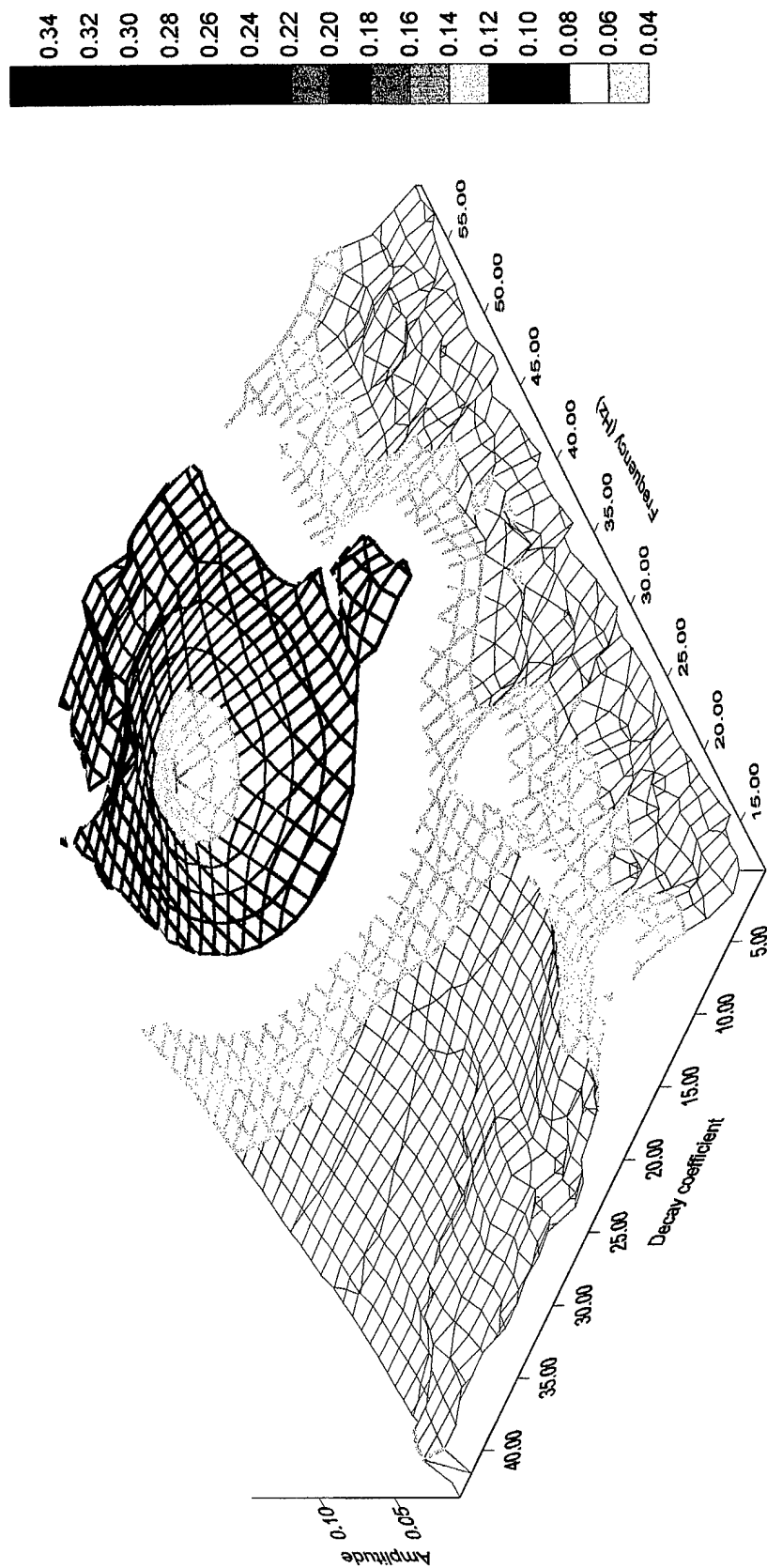


Figure 4a. Subject 3, parietal, thought stimulus = 7

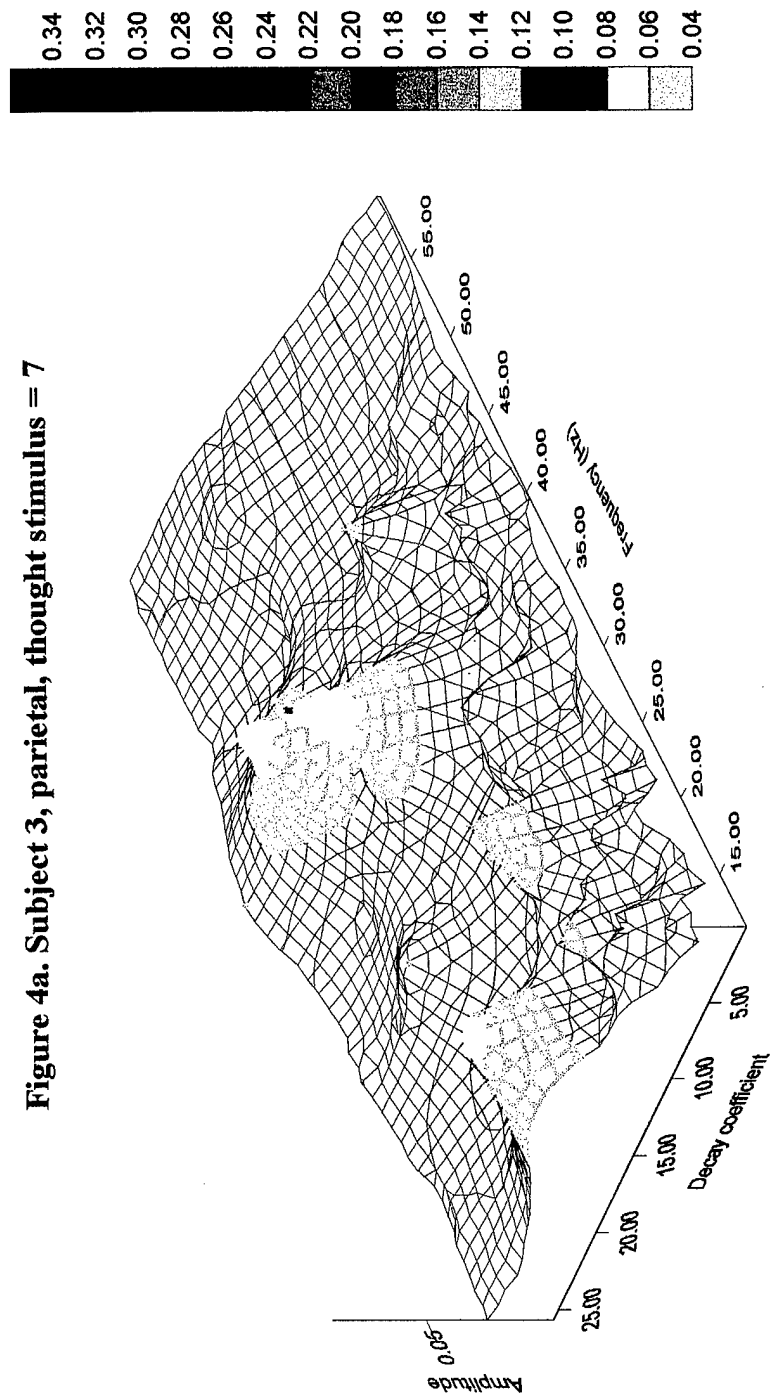


Figure 4b. Subject 3, occipital, thought stimulus = 7

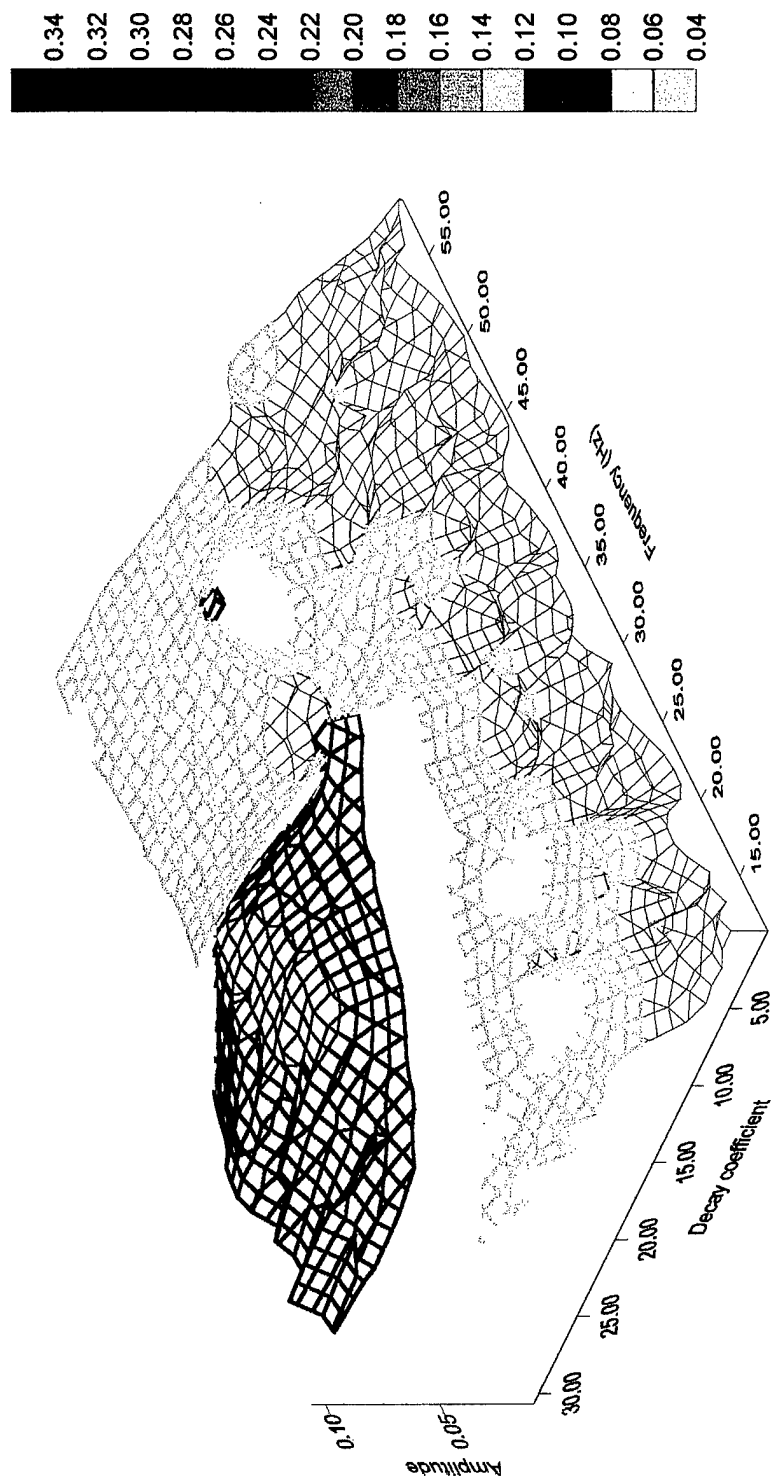




Figure 5a. Subject 1, parietal, thought stimulus = 7

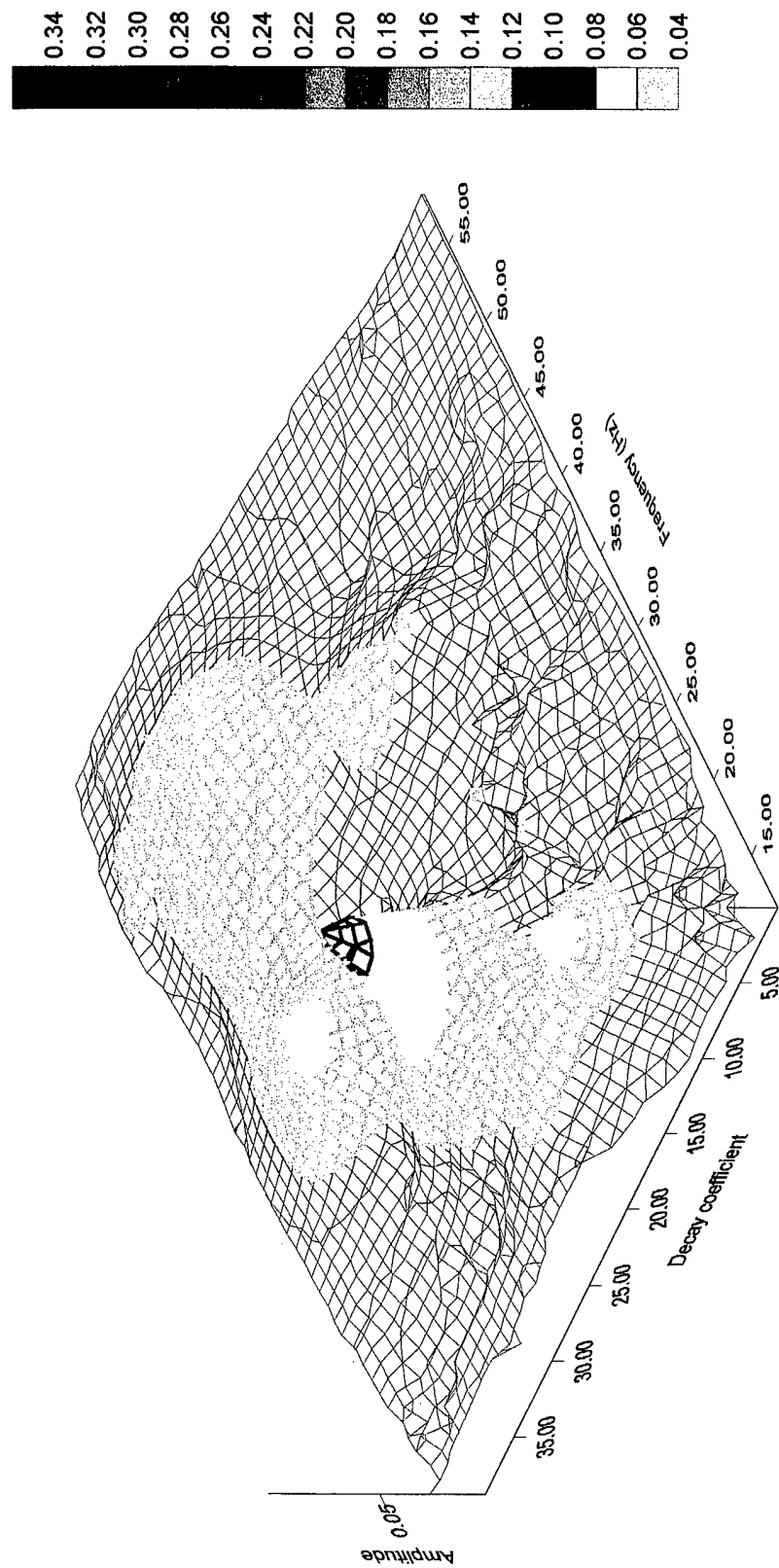


Figure 5b. Subject 1, parietal, image stimulus = 7

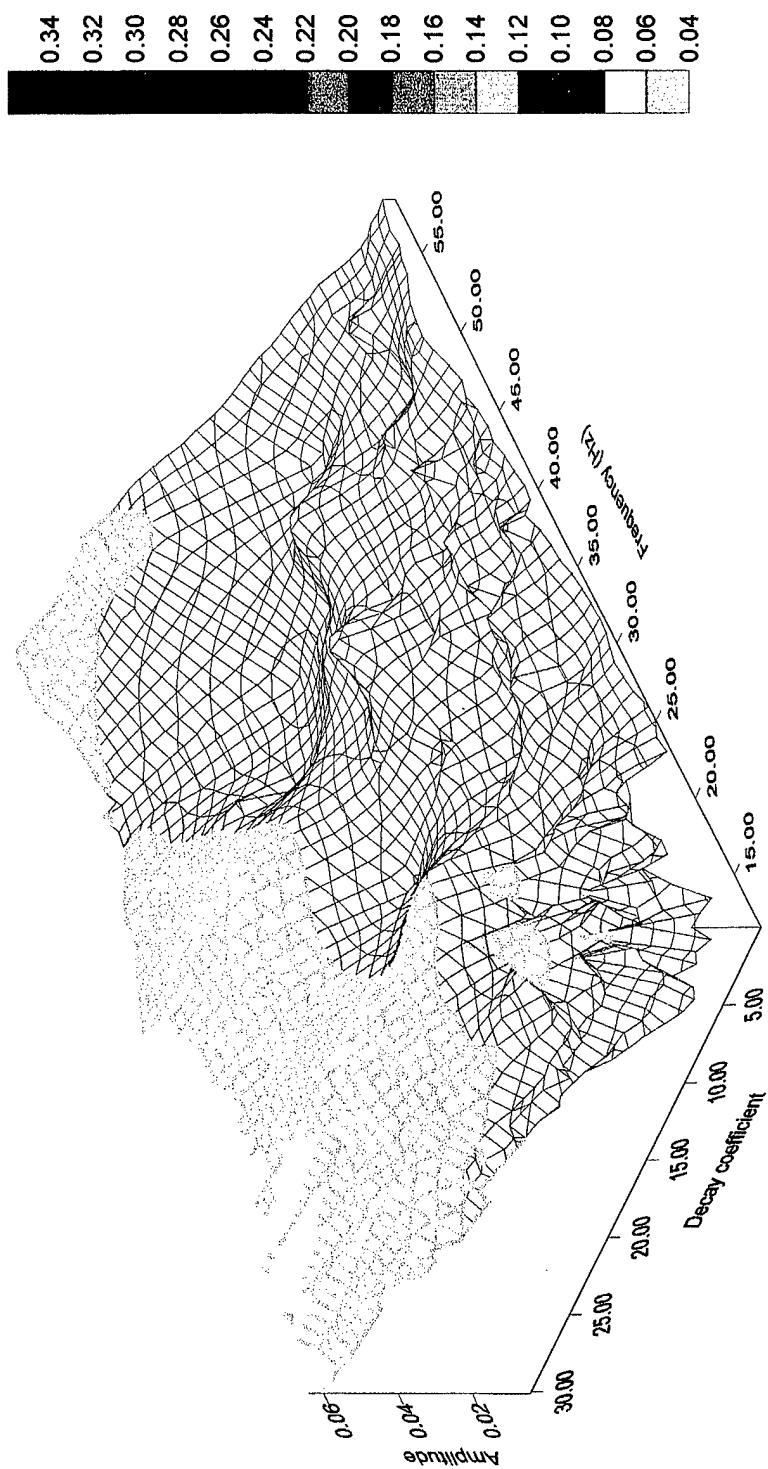


Figure 5c. Subject 1, occipital, thought stimulus = 7

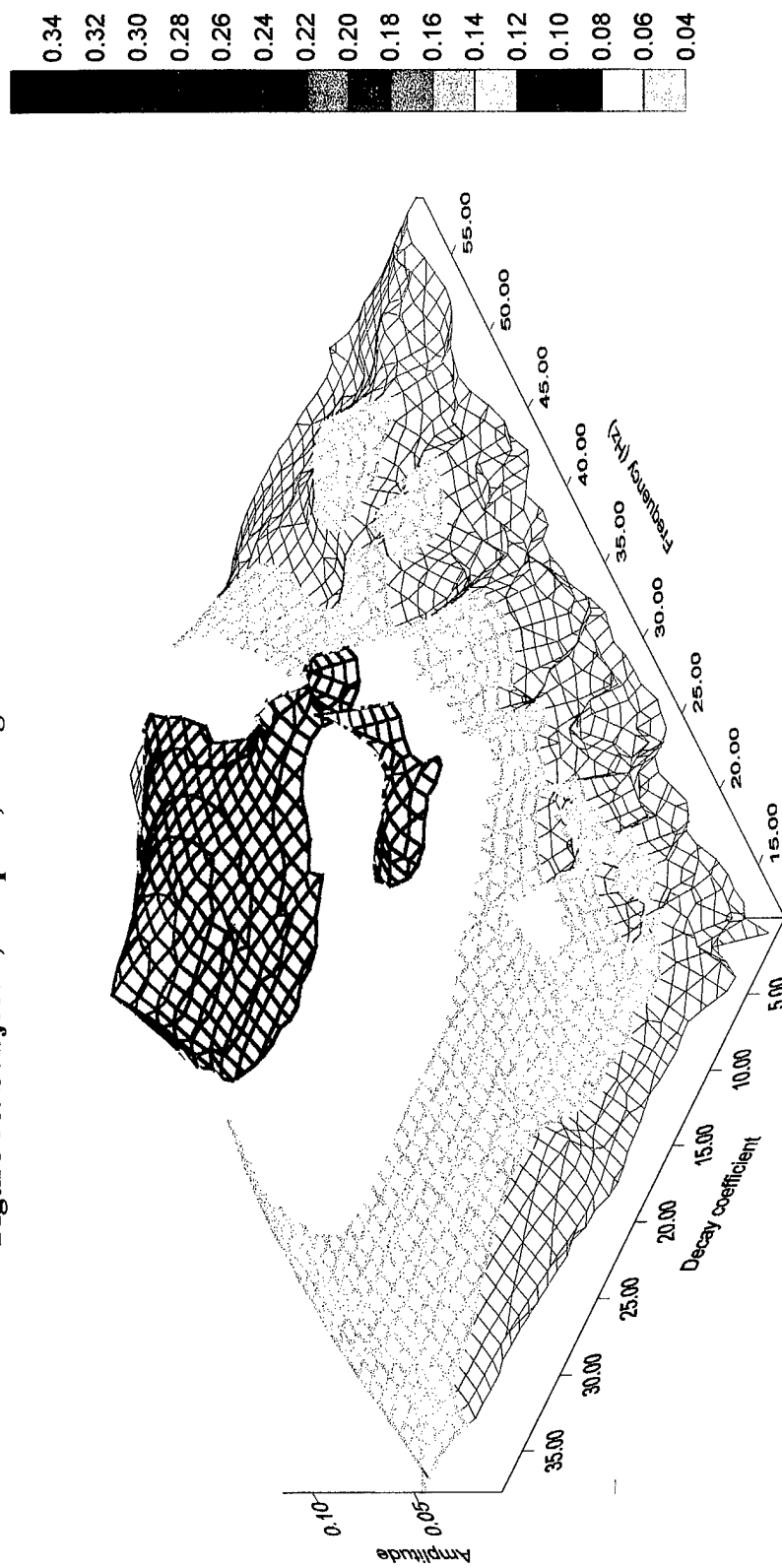


Figure 5d. Subject 1, occipital, image stimulus = 7

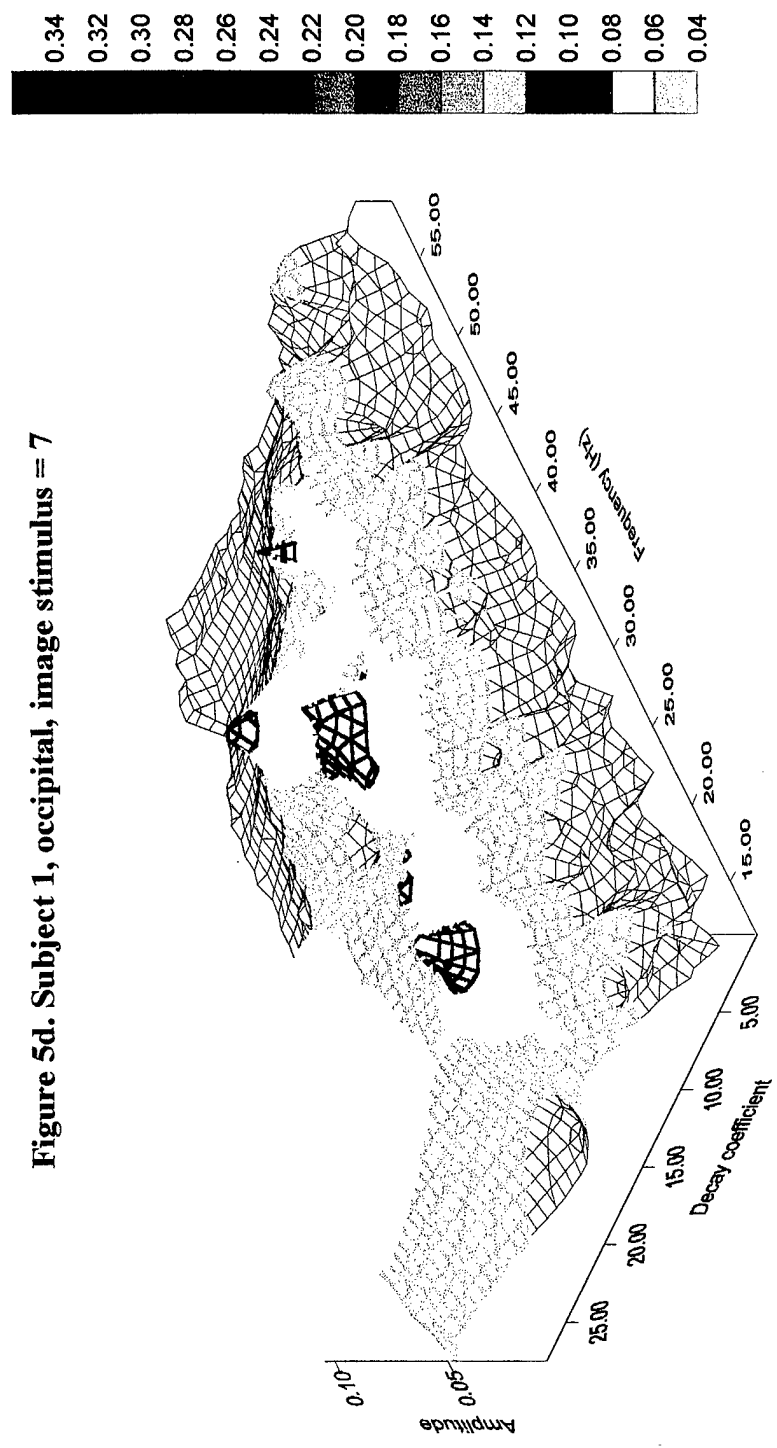


Figure 6a. Subject 1, occipital, thought stimulus = 2

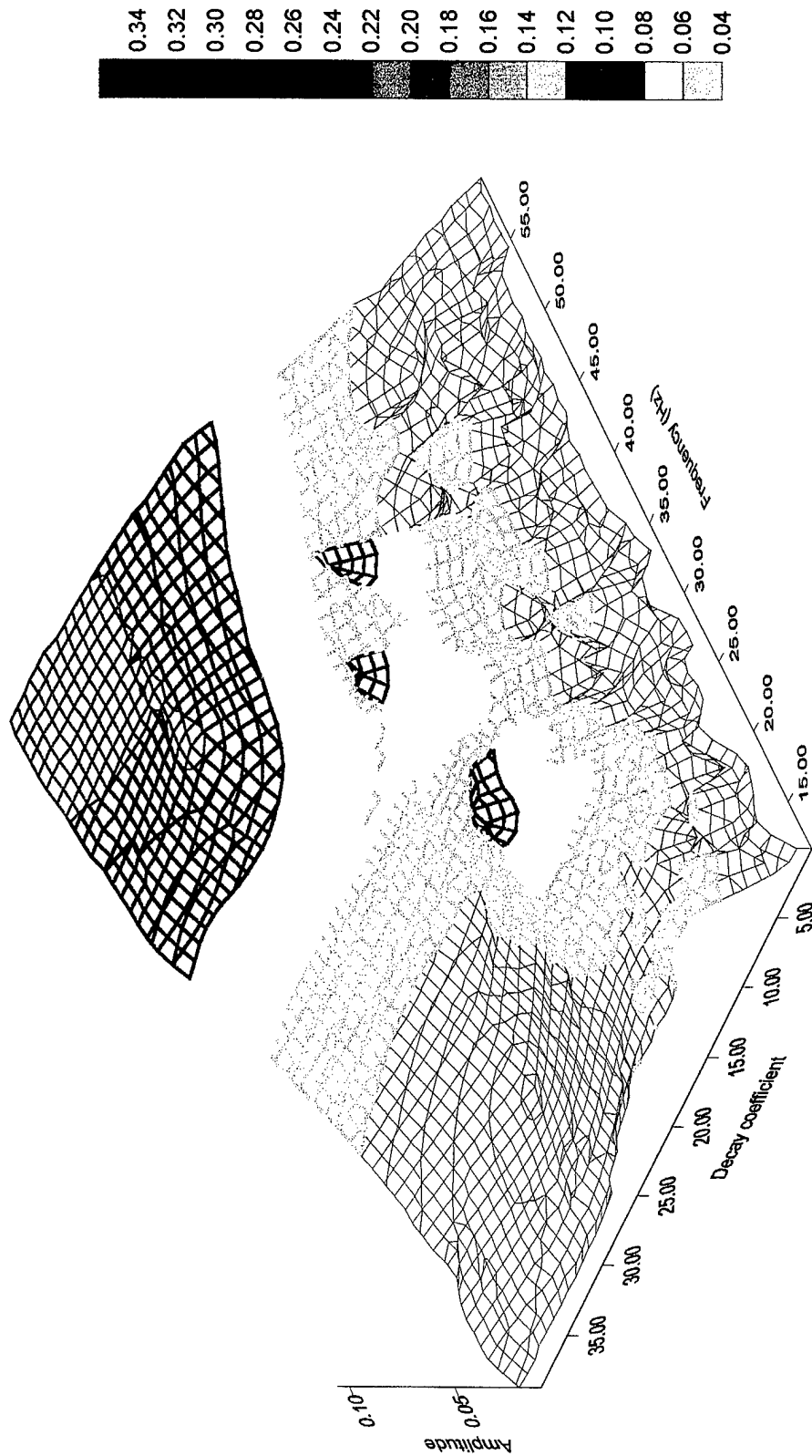


Figure 6b. Subject 1, occipital, image stimulus = 2

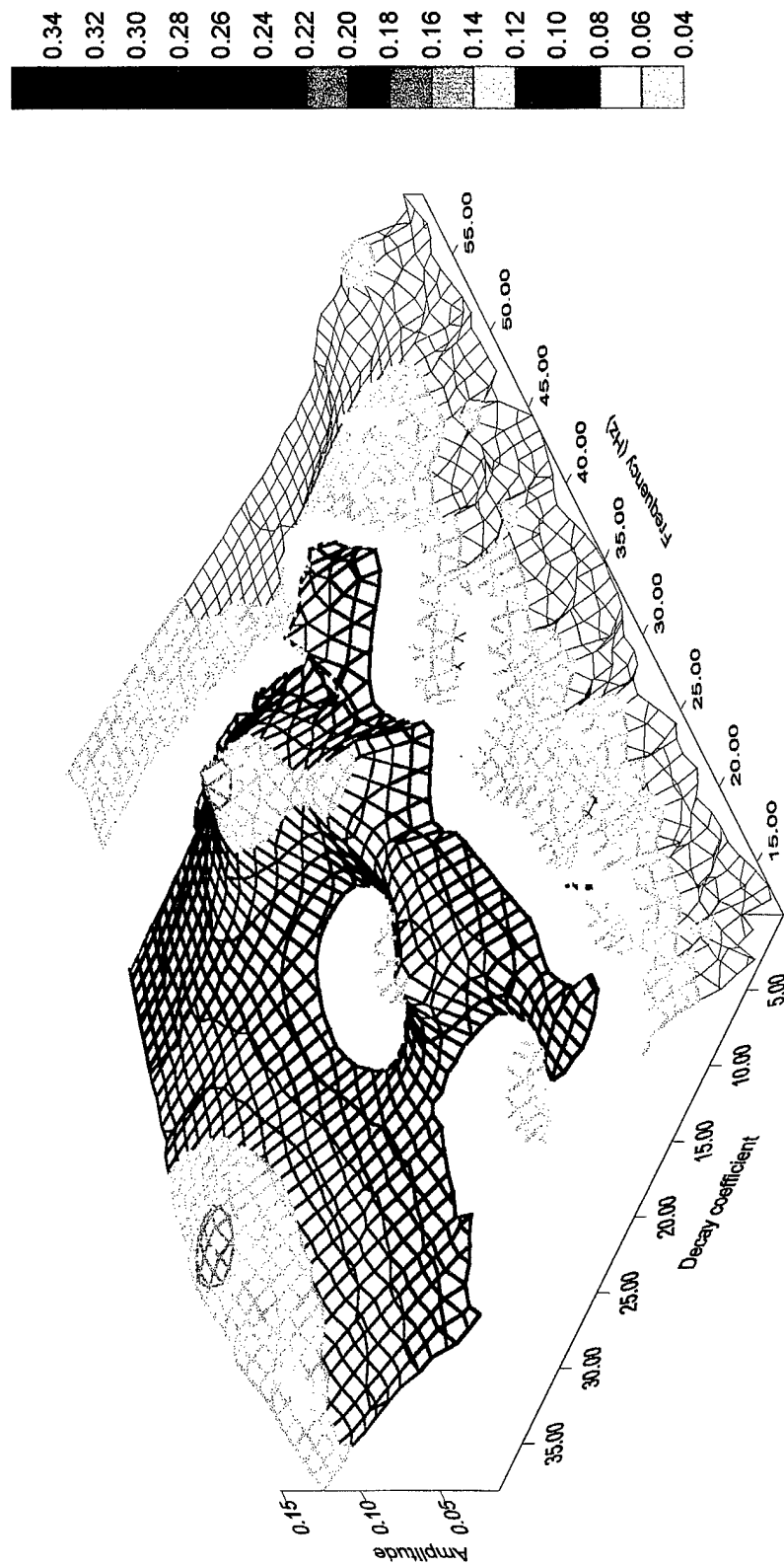


Figure 7a. Subject 2, occipital, thought stimulus = 2

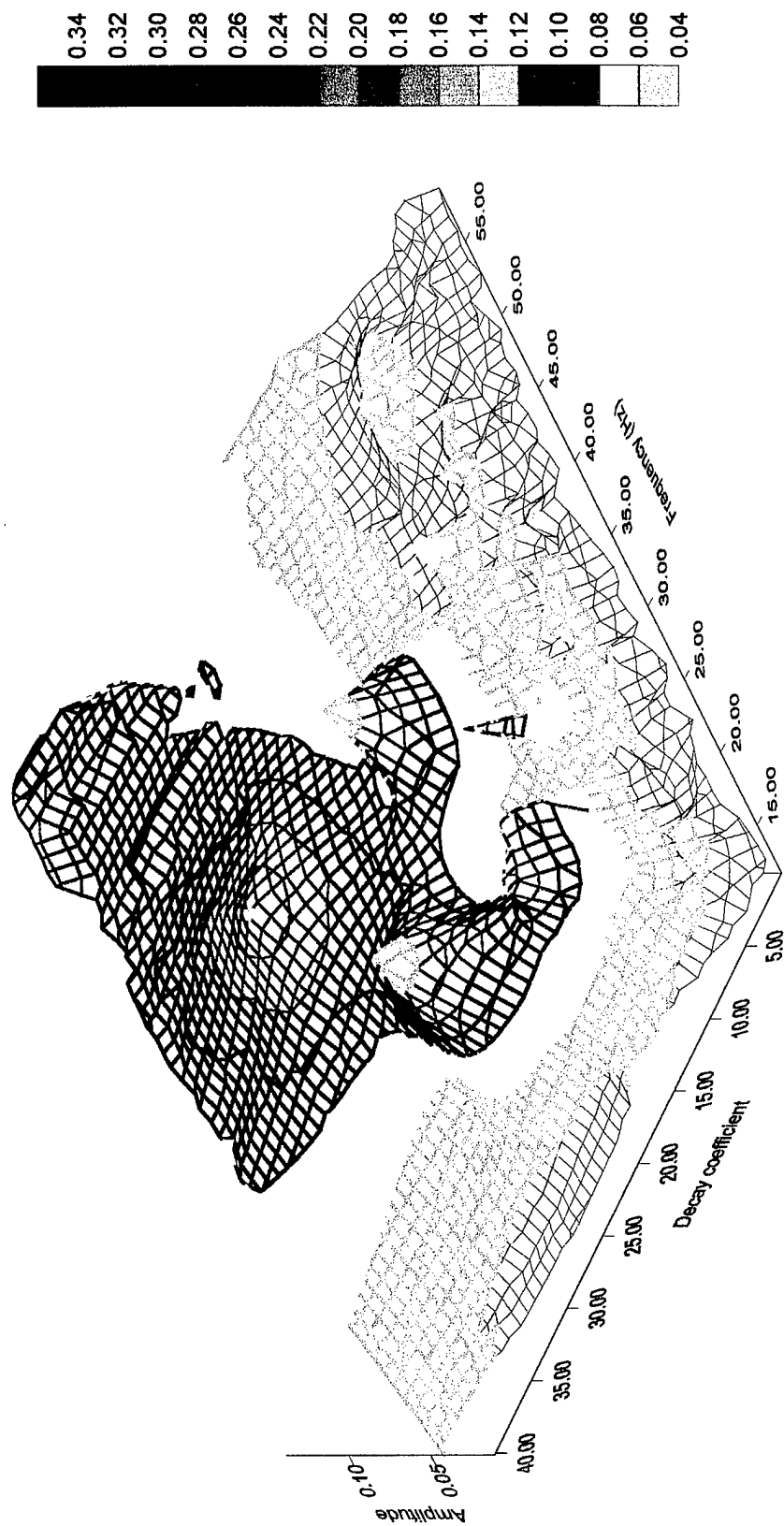


Figure 7b. Subject 2, occipital, image stimulus = 2

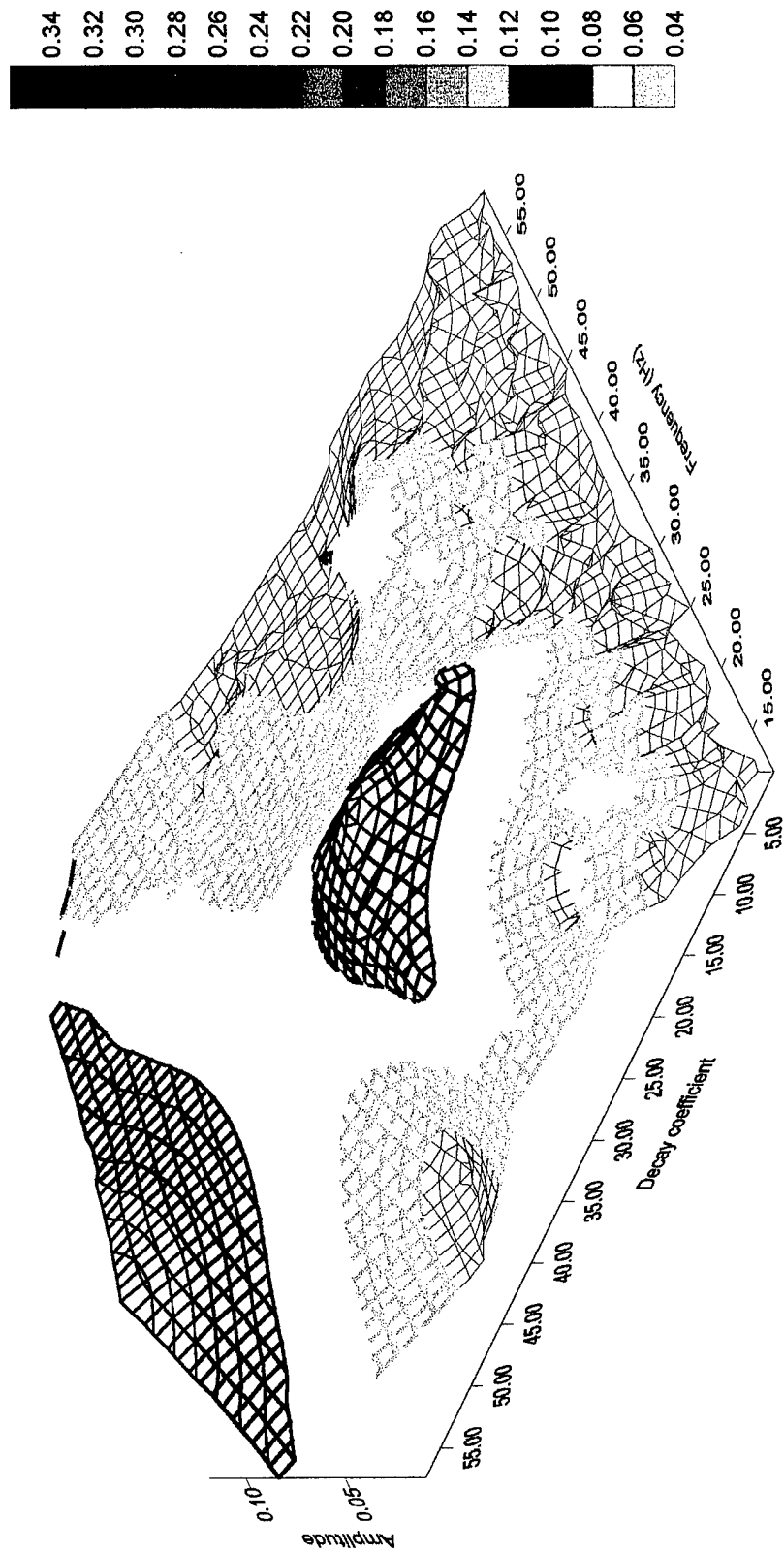




Figure 8a. Subject 3, parietal, thought stimulus = 9

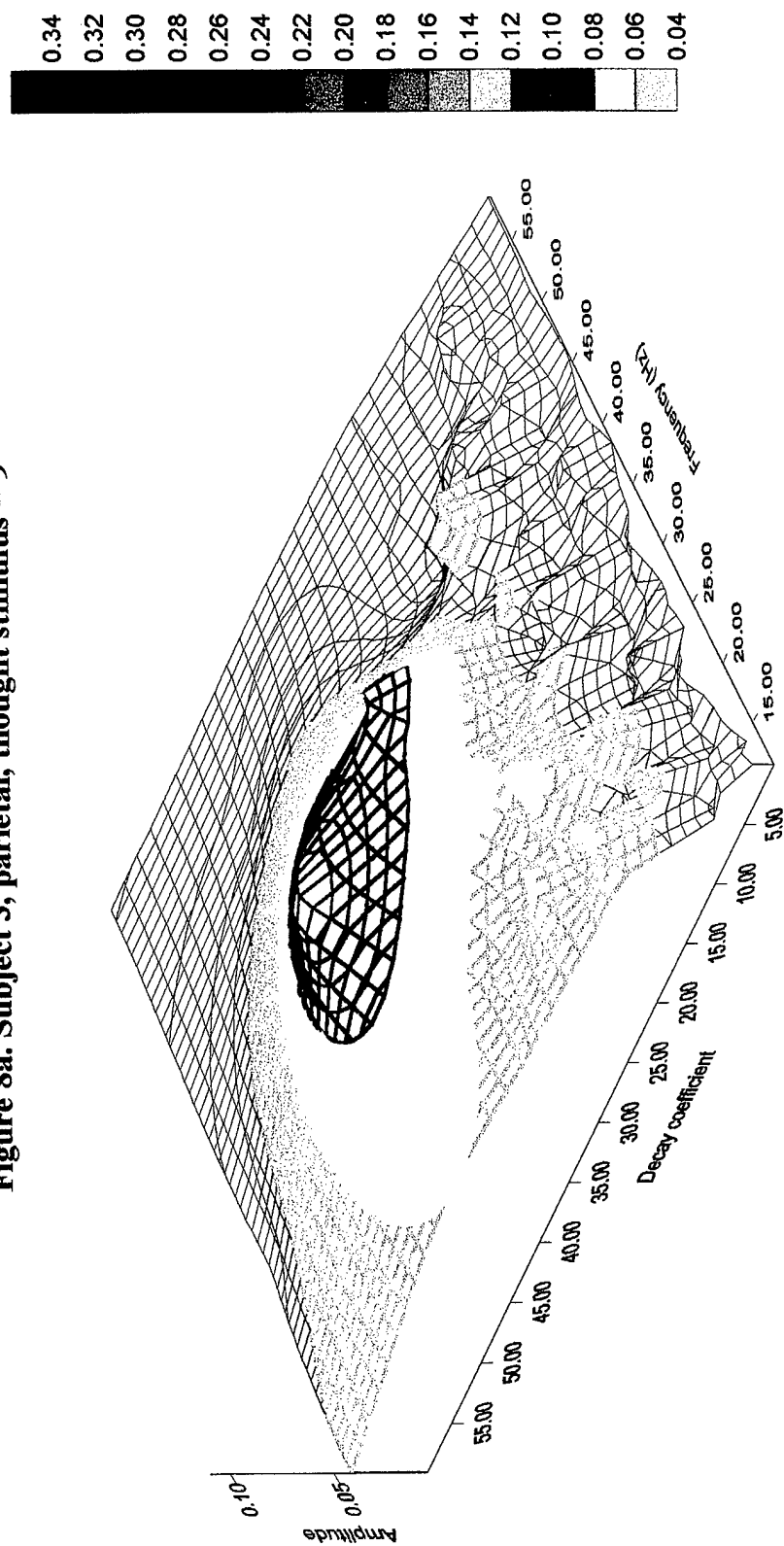
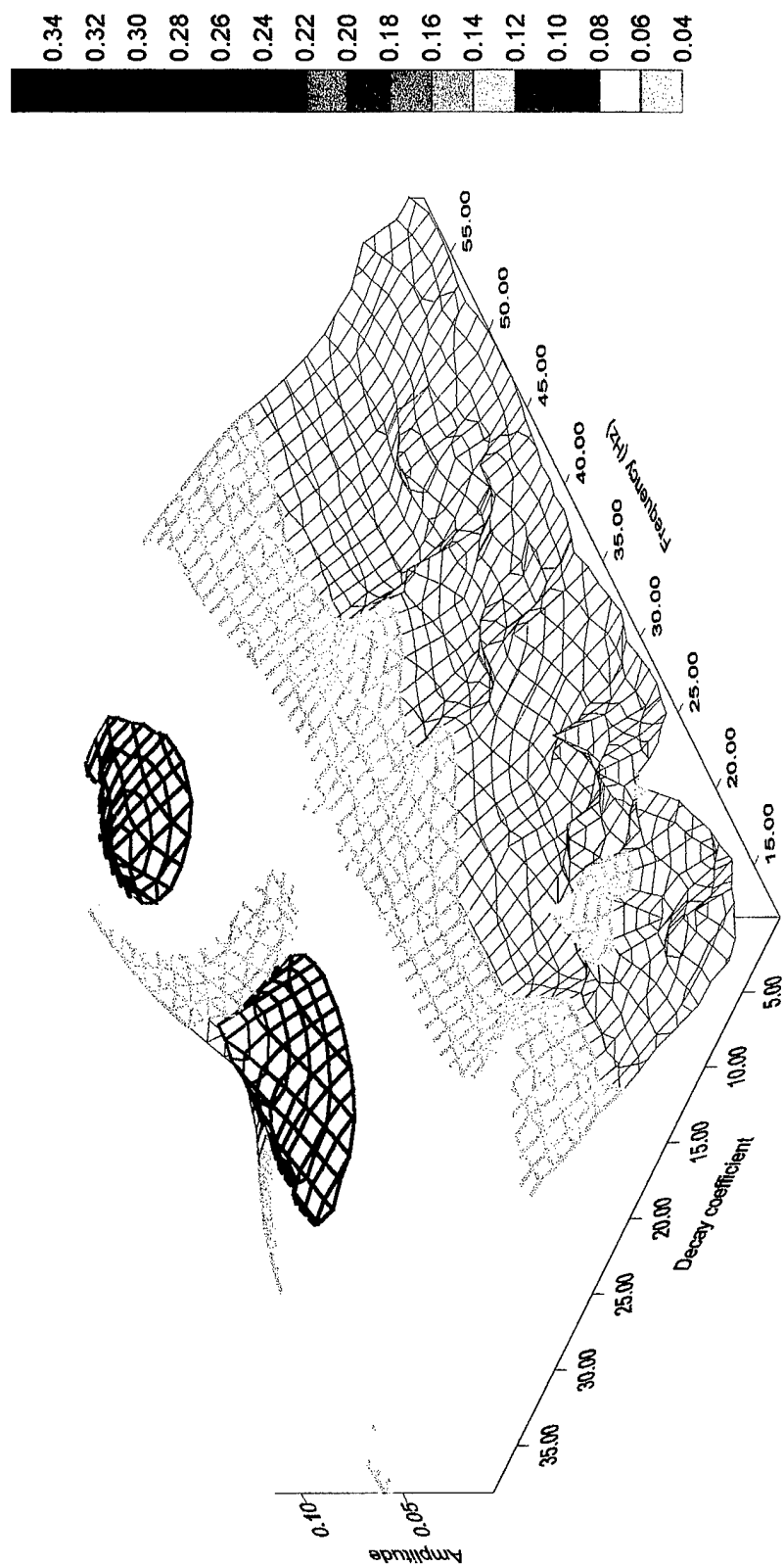


Figure 8b. Subject 3, parietal, image stimulus = 9



the parametric data of specific neural activity for given responses to a cognitive stimulus and relates the frequencies of the signals activated during cognition, their intensities (including phase information), and the rate at which they decay. The analytical data from approximately thirty epochs of the same stimulus type were sufficient to establish a stable, unique map for each cognitive stimulus, electrode location, and test subject.

Figures 1 through 7 show examples of 3-D surface maps described above. In each of the figures the x-axis is the frequency in Hertz, the y-axis is the decay coefficient, and the z-axis is the amplitude (including phase). Figures with the label 'parietal' in their title correspond to the information obtained from scalp electrodes placed over Brodmann's area 39/40, left hemisphere. Similarly, figures with the label 'occipital' in their title correspond to electrodes over Brodmann's area O<sub>2</sub>, the right hemisphere occipital placement.

The primary purpose and result of the experiments described here, supported by the computational analyses, together with the 3-D plots, is the demonstration that individual characteristic neural activity is obtained for specific thought stimuli, electrode locations, and different human test subjects. All experiments provided characteristic 3-D maps for all the experimental conditions. They are stable maps; the topological surface features of each map do not change with increased amounts of data from further acquisition of experimental epochs for each thought stimulus. It should be remembered that these maps are the result of a multi-step process of abstraction based on a specific computational model for cognitive processing described earlier. Each map comprises about 300 data points (on the average) representing the decomposition parameters for the subwaveforms for each of the approximately 30 epochs acquired of the neuroelectric activity associated with a given stimulus. Thus each map is an abstract representation of the neural activity involved in cognitive processing of a specific thought stimulus at a specific electrode location. These parametric maps will form the sufficient basis set of information that will enable a direct identification scheme for eventual real time brainwave computer control applications.

As a matter of secondary interest, there are a number of scientific questions that can be asked regarding neural activity in cognitive processing with the type of experiments described here and for future experiments. As an example, one question immediately and always asked in discussing the neural activity maps and cognitive processing is whether there is evidence for similar neural activity between people thinking of the same linguistic items. The presumption in the question is that even if totally independent people have different specific brain circuitry and language experience, there has to be some fundamental process/mechanism that enables people, in principle, to communicate, and perhaps it might be similar neural activity for the concepts and words used. A second question has to do with the possibility of similar cognitive activity in a given individual when thinking of a linguistic item versus responding to an image of it. There is some suggestive evidence in the neural activity map data that both these questions could be addressed seriously with further refinements in experimental protocol, while still

addressing the primary issue of employing real time brain neural activity during cognition to direct computer functions.

### **Discussion**

The results represent a minimum pilot study in which EEG ERP data were acquired and analyzed for cognitive processing characteristics related to thinking of the numbers zero to nine and the words 'yes' and 'no'. The model considerations leading to the experiments have been described above. Stable and characteristic neural activity maps were predicted to be found, and were found, for thought stimuli retrieved from memory, just as they have been for responses to linguistic stimuli presented on a computer monitor screen (present experiment, and earlier ones yet to be reported). These results comprise a first step in the possibilities for developing a direct brain to computer interface based on real time neural activation related to cognitive processing of specific information. The 3-D maps of abstract parametric data describe a range of neural activity associated with specific linguistic items. The idea is that when such a map is stored in a computer memory, a subsequent single epoch of neural activity associated with the same linguistic item can be identified as belonging to that same map, given an analysis of the ERP. The identification of the map can then point to the ASCII word correlate for immediate further use.

The individual epoch ERPs for memory retrieval items were just as robust as those responses to images of the same linguistic items on a computer monitor. Although time zero ( $t_0$ ) was known precisely for the image stimuli experiment,  $t_0$  identification for the memory retrieval experiment posed a problem. Acquired data after the pixel cross lit up as a time marker on the computer monitor failed to reveal any obvious indication of the start of cognitive processing. More work will have to be done on this problem in future experiments as no good, obvious, and direct method was found to solve the problem for  $t_0$  identification. An indirect method served for the present study. Calculation of a time average of the data for all the thought stimuli events for any given stimulus number for a given test subject and electrode location, followed by comparing that waveform with the time average waveform for the events produced by responding to an image of the same stimulus number, demonstrated that the two waveforms matched in some time history features for the region between 100 and 200 milliseconds. This was true for all stimuli and electrode locations. Since the image response  $t_0$  was known, the thought stimulus  $t_0$  could be inferred to be approximately the same, given the time history matching features. On this basis, a uniform  $t_0$  starting point was chosen for all thought stimuli events.

Intercomparisons between Figures 1a, 1b, and 1c show a similar topological feature among three subjects in the band of neural activity corresponding to values of the decay coefficient from 10 to 25. (Note that in these and subsequent figures the y-scales are often different in different plots dependent on data outcomes from different experiments). The plot for each subject for the thought stimulus '2' is characteristic for that individual, but there is a similar 3-D surface feature in the region near the coordinates for 30 Hz and a decay coefficient of 25 for all three subjects. The feature and surrounding topological

region suggests that there may be something common in the neurological activity regarding cognitive processing of the number '2' among separate individuals. Again, there are suggestions for common topological features in Figures 2a and 2b, for two individuals in the instance when the thought retrieval stimulus is '9'. Both Figures 1 and 2 are for the parietal electrode location. Figures 3a, 3b, and 3c show the data for the thought stimulus '9' at the occipital electrode location for the three subjects. Again, there are hints of common features to look for in more refined experiments. Remember there is no visual stimulus of the number '9' involved, but the visual cortex is showing the evidence for cognitive processing of the thought stimulus.

In Figures 4a and 4b, there are suggestions of common features for a given subject for both the parietal and occipital location when the thought stimulus is '7'. This was a rare occurrence in this preliminary experiment for different thought stimulus numbers and subjects, but has been observed before in image stimulus experiments.

Figures 5a and 5b, for a given subject, show the parietal neural activity data of the thought stimulus experiment versus the image stimulus experiment when the stimulus number is '7'. Figures 5c and 5d do the same for the occipital neural activity data. Given any reasonable feature correlations, the question is whether cognitive processing of a thought stimulus has something in common with the cognitive processing of an image stimulus, or alternatively, whether perhaps cognitive representation in memory of a linguistic item is always simultaneously involved as a major factor in real time cognitive processing. Figures 6, 7, and 8 show more comparison data for thought and image stimulus experiments.

The analytic approach used here yields frequencies that are not harmonics and describes only the pulse-coded behavior in the original signal while ignoring everything else as background noise. Each analytic result is an exact solution of the specified number of simultaneous equations and conjugate pole pairs in the experimental data. As implemented, the exact solution is required to meet some explicit mathematical criteria, and when this is completed, the output lists the parameter values for the irregular basis set. The irregular basis set describes each of the simultaneous decomposition waveforms in terms of their distinct frequency, amplitude, phase, and decay characteristics.

The group waveform set as described above provides a powerful and quantitative analytic approach to macroscopic electrical signaling produced by the brain in response to cognitive stimuli. As noted, the nature of the ERP signature has been presumed to be entirely different in structure than has been classically described. In the most general sense, ERP signatures have previously been described and analyzed as if they were unitary signals in which sequential brain events during cognitive processing produced consequent and reproducible time-specific components. The present work does not make the assumption that the ERP is a single unitary signal, but rather that it is a composite signal that can be decomposed into a finite set of individual waveforms. These waveforms and their parametric coefficients, then, would be the analytic quantities, rather than the classically named N and P components, that an investigator would track in

cognitive experiments that manipulate physiological or psychological variables as part of the experimental design. The reconstruction of the ERP signature from the irregular decomposition basis set shows that N and P components (such as the P3) are artifacts of the summation process involving the waveforms.

In a few instances, there was a virtual repeat of time domain waveform characteristics starting at around 350 milliseconds. It was not a consistent observation, but it might become more common with improved experiments. All processed data used a time window corresponding to the time period between 50 and 562 milliseconds after the designated  $t_0$ .

In future experiments, there will be an endeavor to improve experimental procedures and computer techniques for data acquisition in thought retrieval experiments. In the present experiment the subject's attention was engaged in the time between the recording of the previous item and the end of the random time interval between 2.5 to 3.5 seconds before the pixel cross lit up on the computer monitor. As reported by the test subjects, the attention requirement introduces some element of stress and fatigue over the experimental session. One possibility to avoid this problem may be to have the subject think sequentially of three or four numbers in a row, then report them. The data acquisition scheme should be designed clever enough to pick up ambient data 500 msec. or so before the thought retrieval of the first number. The current procedure is also open to the criticism that the subjects may have rehearsed the items to be thought of prior to the pixel signal appearance, or otherwise engaged unconsciously other simultaneous cognitive processing mechanisms, given the instruction to be prepared to think of an item from the stimulus set when the pixel signal appeared. The new approach should help reduce possible contamination from these sources and be more like free flowing thought production.

### **Experimental Design and Procedures**

The principal experiment was designed to capture physiological voltage signals associated with neocortical information processing as memory retrieval of digits and words took place in a sequence determined by the test subject. A small computer-generated cross formed from a few pixels was presented in the center of a computer monitor screen and was the signal for the test subject to think of a digit from zero to nine or the word 'yes' or 'no'. After the thought was completed, it was reported to the person handling the data acquisition who then entered the reported item into the computer. In this manner the sequence and the number of items in each of the twelve categories were archived for each experiment. The subjects were instructed to be prepared to think of a number or word from the target list of twelve items as soon as the pixel cross was lit. The instruction was intended to limit the time period in which it would be necessary to search for indications of the initiation of cognitive processing (or time zero) associated with the thought activation.

The secondary experiment was designed to elicit physiological voltage signals associated with neocortical information processing in response to displayed digits and words on a computer monitor from the same set of items used in the primary experiment.. A special computer-generated presentation was prepared in which randomly ordered and randomly spaced, yet precisely timed, stimuli appeared to the experimental subjects on a computer monitor. The randomness was achieved by use of a standard mathematical card shuffling routine adapted for the purpose of this experiment. The precise timing enabled correlation with the beginning of the target physiological signal. Each of the twelve stimuli appeared forty times. The stimuli also had random continuous perturbations of plus or minus a half second around the average stimulus separation time of 3 seconds. The test subject was allowed to temporarily halt the session for a break whenever it was requested.

Six electrodes were placed on each subject at frontal, parieto-temporal, and occipital locations over both hemispheres, corresponding to Brodmann's areas 46 and 39/40, plus occipital placement  $O_1$  on the left hemisphere, and mirror image placements on the right hemisphere. Each electrode measurement on a given hemisphere was referenced to a mastoid process electrode location on that side of the head. All recordings are of unipolar signals. The experimental protocol was an instance in which the subjects were informed and trained subjects, not naive individuals. Each subject was fully informed about the design of the experiment, its purposes, and the procedures to be followed. A personal computer (PC) was used in the preparation and presentation of the pixel timing signal and for the image stimuli. The subjects spent up as much time as needed with a test computer file that familiarized them with the stimuli images, and provided an opportunity to receive training for the short duration interval (600 msec.) of each image stimulus presentation.

A computer-controlled, multichannel Grass EEG test instrument was used to take the EEG data, that were then digitized at a 1 KHz clock rate by a Keithley Instruments analog to digital (A/D) converter, sent to the PC, and finally stored on a hard drive. Over 1.1 seconds of data were recorded in this fashion for each of the six data channels at the onset of each thought, or stimulus presentation, and are permanently archived.

The data analysis simultaneously extracted the frequency, phase, and time-domain information content from the ERP signature and presented the output in the form of a basis set of coefficients that describe the decomposition of the signature into constituent subwaveforms. Each of these subwaveforms is a complex exponential function with a distinct amplitude coefficient, unique frequency, phase, and decay coefficient. Before analysis was undertaken, each waveform acquired in the experiment was examined for artifacts disruptive to analytical processing, such as occur in muscle movement, stress, eye blinks, and electrical anomalies. Waveforms with artifacts that would preclude successful processing were removed from the data set.

### **Waveform Decomposition Analysis**

The basic assumptions regarding the manifestation of the physiological voltage signal are as given earlier. Additionally, however, the parametric method now being described assumes that the signal contains pulse-coded behavior due to the brain's response to a semantic stimulus presented during a special experimental procedure. Under these conditions, the brain's response has been shown to reflect unique information generating events, and the analysis method shown to decompose successfully the composite signal and provide a description of the underlying pulse-coded subwaveforms (Dickhaut, 1988).

The response of an electrically active neural structure to an impulse function as described in the previous paragraph resulted in a finite sum of complex exponentials, for which a technique known as the SingularityExpansion Method (Baum, 1971; Tesche, 1972) can be applied. Similarly, for N uniformly spaced time samples in the signal, it is proposed that the response of the structure can be written as

$$f(t_i) = \sum_{j=1}^n A_j e^{\alpha_j t_i} \quad (1)$$

where A= complex amplitude,

$\alpha$  = complex natural frequency,

$n$  = number of independent exponentials in the data, and

$t_i = (i - 1) \Delta t$ .

It should be noted that the complex natural frequency does not contain the term  $2\pi$  and therefore is a general description not limited just to harmonic behavior.

In the instance in which the transfer function of the response is represented by

$$F(s) = \frac{(s - \gamma_1)(s - \gamma_2) \dots (s - \gamma_{n-1})}{(s - \alpha_1)(s - \alpha_2) \dots (s - \alpha_n)} \quad (2)$$

where

$\gamma_i$  = a zero of the transfer function, and

$\alpha_i$  = a pole or singularity of the transfer function,



the Laplace transform of the transfer function yields equation (1). A partial fraction expansion of equation (2) yields

$$F(s) = \sum_{j=1}^n \frac{A_j}{s - \alpha_j} \quad (3)$$

where  $A_j$  is defined as the residue for the  $j^{\text{th}}$  pole  $\alpha_j$ . Performing an inverse Laplace transform of equation (3) produces equation (1). Thus the complex natural frequencies correspond to poles, and the amplitudes to residues. (Lager et al., 1977).

Returning to equation (1), suppose that a change in variables has been introduced such that  $\mu_k = e^{\alpha_k t}$ . Then if equation (1) is an equality for all values of  $t$ , the equations

$$\begin{aligned} A_1 + A_2 + \dots + A_n &= f_0 \\ A_1\mu_1 + A_2\mu_2 + \dots + A_n\mu_n &= f_1 \\ A_1\mu_1^2 + A_2\mu_2^2 + \dots + A_n\mu_n^2 &= f_2 \\ &\vdots \\ A_1\mu_1^{N-1} + A_2\mu_2^{N-1} + \dots + A_n\mu_n^{N-1} &= f_{N-1} \end{aligned} \quad (4)$$

are necessarily satisfied, and equation (1) may be based on the results of satisfying these equations as close as is possible (Hildebrand, 1956). The  $\mu$ 's are also to be determined, requiring at least  $2n$  equations, and the equations are nonlinear in the  $\mu$ 's. This difficulty can be minimized by adopting a technique known as Prony's method (Prony, 1795). The following description of Prony's method follows that given by Hildebrand (1956).

Let  $\mu_1, \dots, \mu_n$  be the roots of the algebraic equation

$$\mu^n - \alpha_1\mu^{n-1} - \alpha_2\mu^{n-2} - \dots - \alpha_{n-1}\mu - \alpha_n = 0 \quad (5)$$

so that the left-hand side of equation (5) is identified with the product  $(\mu - \mu_1)(\mu - \mu_2) \dots (\mu - \mu_n)$ . Proceed to modify the equations in (4) by multiplying the first equation by  $\alpha_n$ , the second by  $\alpha_{n-1}$ , ..., the  $n^{\text{th}}$  equation by  $\alpha_1$ , and the  $(n+1)^{\text{th}}$  equation by  $-1$ , then add the results. Using the fact that each  $\mu$  satisfies (5), the result of the addition is in the form

$$f_n - \alpha_1 f_{n-1} - \dots - \alpha_n f_0 = 0. \quad (6)$$

A set of  $N - (n - 1)$  additional equations with a similar form can be obtained in the same manner by beginning instead, successfully, with the second, third, fourth, ...,  $(N - n)^{\text{th}}$  equations. Thus, it is seen that (4) and (5) imply the set of  $N - n$  linear equations

$$\begin{aligned}
 f_{n-1}\alpha_1 + f_{n-2}\alpha_2 + \cdots f_0\alpha_n &= f_n \\
 f_n\alpha_1 + f_{n-1}\alpha_2 + \cdots f_1\alpha_n &= f_{n+1} \\
 &\vdots \\
 f_{N-2}\alpha_1 + f_{N-3}\alpha_2 + \cdots f_{N-n-1}\alpha_n &= f_{N-1}.
 \end{aligned}
 \tag{7}$$

The ordinates  $f_k$  are known, which means this set generally can be solved directly for the  $n\alpha$ 's if  $N = 2n$ . Once the  $\alpha$ 's are determined, the  $n\mu$ 's are found as the roots of (5) and may be real or complex. The equations in (4) then become linear equations in the  $nA$ 's, with known coefficients. The  $A$ 's can be determined from the first  $n$  of these equations. Note that the non-linearity of the system has been concentrated in the single algebraic equation given in (5).

The outline just given provides a summary of the mathematical essentials of the waveform decomposition methodology. The decomposition output comes directly from an exact solution to the specified number of equations describing the input data.

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